

Decomposition of Continuity Using $g^\#$ -Closed Sets

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Abstract -- There are various types of generalization of continuous maps in the development of topology. Recently some decompositions of continuity are obtained by various authors with the help of generalized continuous maps in topological spaces. In this paper we obtain a decomposition of continuity using a generalized continuity called $g^\#$ -continuity in topology.

Key words and Phrases: \tilde{g} -closed set, $g^\#lc^*$ -set, $g^\#$ -continuous map, $g^\#lc^*$ -continuous map.

1. Introduction

Different types of generalizations of continuous maps were introduced and studied by various authors in the recent development of topology. The decomposition of continuity is one of the many problems in general topology. Tong [7] introduced the notions of A-sets and A-continuity and established a decomposition of continuity. Also Tong [8] introduced the notions of B-sets and B-continuity and used them to obtain another decomposition of continuity and Ganster and Reilly [2] have improved Tong's decomposition result. Przemski [6] obtained some decomposition of continuity. Hatir, Noiri and Yuksel [3] also obtained a decomposition of continuity. Dontchev and Przemski [1] obtained some decompositions of continuity. In this paper, we obtain a decomposition of continuity in topological spaces using $g^\#$ -continuity in topological spaces.

2. Preliminaries

Throughout this paper (X, τ) and (Y, σ) (or X and Y) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ) , $cl(A)$

and $int(A)$ denote the closure of A and the interior of A respectively.

We recall the following definitions which are useful in the sequel.

2.1. Definition

A subset A of a space (X, τ) is called:

α -open set [5] if $A \subseteq int(cl(int(A)))$.

The complement of α -open set is said to be α -closed.

2.2. Definition

A subset A of a space (X, τ) is called:

(i) a α -generalized closed (briefly α g-closed) set [4] if $\alpha cl(A) \subseteq U$, whenever $A \subseteq U$ and U is open in (X, τ) . The complement of α g-closed set is called α g-open set;

(ii) a $g^\#$ -closed set [9] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α g-open in (X, τ) . The complement of $g^\#$ -closed set is called $g^\#$ -open.

2.3. Definition [9]

A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be $g^\#$ -continuous if for each closed set V of Y , $f^{-1}(V)$ is $g^\#$ -closed in X .

2.4. Proposition [9]

Every closed set is $g^\#$ -closed but not conversely.

2.5. Proposition [9]

Every continuous map is $g^\#$ -continuous but not conversely.

3. Decomposition of Continuity

In this section by using $g^\#$ -continuity we obtain a decomposition of continuity in topological spaces.

To obtain a decomposition of continuity, we first introduce the notion of $g^\#lc^*$ -continuous map in topological spaces and prove that a map is continuous if and only if it is both $g^\#$ -continuous and $g^\#lc^*$ -continuous.

We introduce the following definition.

3.1. Definition

A subset A of a space (X, τ) is called $g^\#lc^*$ -set if $A = M \cap N$, where M is α g-open and N is closed in (X, τ) .

3.2. Example

Let $X = \{a, b, c\}$ with $\tau = \{\emptyset, \{b\}, X\}$. Then $\{a\}$ is $g^\#lc^*$ -set in (X, τ) .

3.3. Remark

Every closed set is $g^\#lc^*$ -set but not conversely.

3.4. Example

Let $X = \{a, b, c\}$ with $\tau = \{\emptyset, \{c\}, X\}$. Then $\{b, c\}$ is $g^\#lc^*$ -set but not closed in (X, τ) .

3.5. Remark

$g^\#$ -closed and $g^\#lc^*$ -sets are independent of each other.

3.6. Example

Let $X = \{a, b, c\}$ with $\tau = \{\emptyset, \{b, c\}, X\}$. Then $\{a, b\}$ is an $g^\#$ -closed set but not $g^\#lc^*$ -set in (X, τ) .

3.7. Example

Let $X = \{a, b, c\}$ with $\tau = \{\emptyset, \{b\}, X\}$. Then $\{a, b\}$ is an $g^\#lc^*$ -set but not $g^\#$ -closed set in (X, τ) .

3.8. Proposition

Let (X, τ) be a topological space. Then a subset A of (X, τ) is closed if and only if it is both $g^\#$ -closed and $g^\#lc^*$ -set.

Proof

Necessity is trivial. To prove the sufficiency, assume that A is both $g^\#$ -closed and $g^\#lc^*$ -set. Then $A = M \cap N$, where M is α g-open and N is closed in (X, τ) . Therefore, $A \subseteq M$ and $A \subseteq N$ and so by hypothesis, $cl(A) \subseteq M$ and $cl(A) \subseteq N$. Thus $cl(A) \subseteq M \cap N = A$ and hence $cl(A) = A$ i.e., A is closed in (X, τ) .

3.9. Definition

A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be $g^\#lc^*$ -continuous if for each closed set V of (Y, σ) , $f^{-1}(V)$ is a $g^\#lc^*$ -set in (X, τ) .

3.10. Example

Let $X = Y = \{a, b, c\}$ with $\tau = \{\emptyset, \{a\}, X\}$ and $\sigma = \{\emptyset, \{a\}, \{b, c\}, Y\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity map. Then f is $g^\#lc^*$ -continuous map.

3.11. Remark

Every continuous map is $g^\#lc^*$ -continuous but not conversely.

3.12. Example

Let $X = Y = \{a, b, c\}$ with $\tau = \{\emptyset, \{b\}, X\}$ and $\sigma = \{\emptyset, \{b\}, \{a, c\}, Y\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity map. Then f is $g^\#lc^*$ -continuous map. Since for the closed set $\{b\}$ in $(Y,$

σ), $f^{-1}(\{b\}) = \{b\}$, which is not closed in (X, τ) , f is not continuous.

3.13. Remark

$\mathcal{G}^\#$ -continuity and $\mathcal{G}^\#lc^*$ -continuity are independent of each other.

3.14. Example

Let $X = Y = \{a, b, c\}$ with $\tau = \{\emptyset, \{a, b\}, X\}$ and $\sigma = \{\emptyset, \{a\}, Y\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be the identity map. Then f is $\mathcal{G}^\#$ -continuous but not $\mathcal{G}^\#lc^*$ -continuous.

3.15. Example

Let $X = Y = \{a, b, c\}$ with $\tau = \{\emptyset, \{b\}, X\}$ and $\sigma = \{\emptyset, \{b, c\}, Y\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be the identity map. Then f is $\mathcal{G}^\#lc^*$ -continuous but not $\mathcal{G}^\#$ -continuous.

We have the following decomposition for continuity.

3.16. Theorem

A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is continuous if and only if it is both $\mathcal{G}^\#$ -continuous and $\mathcal{G}^\#lc^*$ -continuous.

Proof

Assume that f is continuous. Then by Proposition 2.5 and Remark 3.11, f is both $\mathcal{G}^\#$ -continuous and $\mathcal{G}^\#lc^*$ -continuous.

Conversely, assume that f is both $\mathcal{G}^\#$ -continuous and $\mathcal{G}^\#lc^*$ -continuous. Let V be a closed subset of (Y, σ) . Then $f^{-1}(V)$ is both $\mathcal{G}^\#$ -closed and $\mathcal{G}^\#lc^*$ -set. As in Proposition 3.8, we prove that $f^{-1}(V)$ is a closed set in (X, τ) and so f is continuous.

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